# Niched-Pareto Genetic Algorithm for Aircraft Technology Selection Process

Chirag B. Patel* Dr. Michelle R. Kirby†

Prof. Dimitri N. Mavris‡

*Aerospace System Design Lab, Georgia Institute of Technology, Atlanta, GA 30332, U.S.A.*

**Design of any complex system entails many objectives to reach and constraints to satisfy. This multi–objective nature of the problem ensures that the technology solution is always a compromise between conflicting objectives. The purpose of this paper is to demonstrate the application of Niched Pareto genetic algorithm as a relatively fast and straightforward method for obtaining technology sets that are distributed along the Pareto frontier in objective space. In this implementation, the genetic algorithm is wrapped around a technology evaluation environment to efficiently evaluate various technology combinations. Some of the major challenges include formulation of Pareto domination tournament and sharing function of Niched Pareto genetic algorithm for a technology selection problem, extracting Pareto front from population of the final generation and visualizing the results.**

## Nomenclature

| | |
|---|---|
| $GA$ | Genetic Algorithm |
| $NPGA$ | Niched Pareto Genetic Algorithm |
| $MODM$ | Multi–Objective Decision Making |
| $DM$ | Decision Maker |
| $TIES$ | Technology Identification Evaluation and Selection |
| $TCM$ | Technology Compatibility Matrix |
| $TIM$ | Technology Impact Matrix |
| $RSE$ | Response Surface Equation |
| $N$ | Population Size |
| $t$ | Number of Technologies |
| $n$ | Number of Objectives |
| $\sigma_{sh}$ | Niche Radius |
| $A_{pareto}$ | Area of the Pareto Front |
| $A_{max}$ | Maximum Possible Area of the Pareto Front |
| $A_{min}$ | Minimum Possible Area of the Pareto Front |
| $M_i$ | Maximum Value of $i^{\text{th}}$ Objective in a Population |
| $m_i$ | Minimum Value of $i^{\text{th}}$ Objective in a Population |

## I. Introduction and Motivation

Technology selection is an important step in designing any advance complex system. When the system is not feasible and/or viable, new technologies may be infused in the system to meet performance requirements and economic constraints. New technologies should be selected on the basis of their impact on the performance and economic parameters in consideration. With addition of each available technology

*Graduate Research Assistant, Student Member AIAA

†Research Engineer II, Member AIAA

‡Director and Boeing Professor of Advanced Aerospace Systems Analysis, Associate Fellow AIAA

option there is a geometric increase in the solution space, which is referred to as *curse of dimensionality*. As a result of this, an exhaustive combinatorial search becomes impractical when the number of technologies are more than about 15 or 20. To overcome this difficulty, a method has been developed, that combines a GA with the TIES process.[1] This method uses GA to select a population of different technology combinations for each generation and uses the TIES process to evaluate the objectives and constraints; much of the basic theory behind this method has been described extensively by Roth and Patel.[2] The main drawback of this method in its present configuration is that the GA tends to give a point converged solution depending on the importance (weighting) assigned to the objective functions. This solution lies on a Pareto frontier. A technology Pareto frontier is defined as the locus of non-dominated optimum technology solutions in the objective space. A point converged solution on two dimensional objective space as achieved by application of canonical GA is depicted in Figure 1 along with a notional Pareto frontier.

It is often desirable to know how the optimal technology combinations vary with change in the requirements, i.e., in the objective weighting. In other words, one would like to see the entire Pareto front populated with non-dominated technology combinations. When the objectives consist of metrics related to cost and risk, it is very important to visualize the behavior of technology combinations with changing constraints. A set of technology combinations on the Pareto frontier would allow the decision makers at the conceptual design phase to intuitively make tradeoffs among the objectives. A notional plot with technology combinations distributed along the Pareto frontier is depicted in Figure 2. One way to obtain the frontier is to use iterative Pareto front technique as suggested by Roth et. al.[3] In this method the technology sets are obtained along the Pareto frontier by parametrically varying the objective weights over a series of GA optimization runs. This method is computationally very expensive and the the quality of Pareto frontier depends on the step size one considers for varying the weighting for every optimization run. Although its application yields explicit information relating an optimal technology set to a specific objective weighting, it becomes very time consuming when the number of objectives to be tracked increase beyond about 10. This paper will demonstrate the generation of Pareto front technology solutions using Niched Pareto GA. The resulting data is then transferred to a visualization software to create an environment where the user can view the data in the form of 3 dimensional plots of selected objectives and correlate the points on the Pareto front with corresponding technology combination and the objective values.

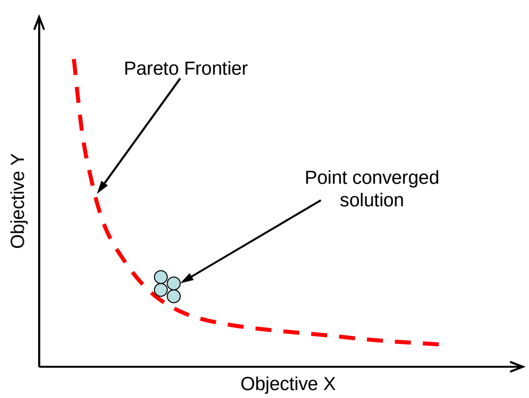


**Figure 1. Notional Pareto Frontier with Point Converged Solution**

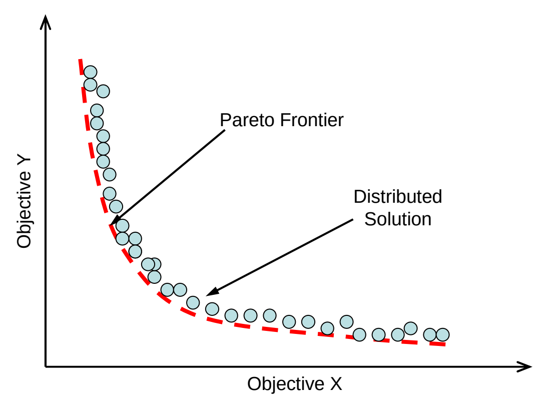


**Figure 2. Notional Pareto Frontier with Final Solution Set Distributed Along the Pareto Front**

### A. Technology Identification, Evaluation and Selection with Genetic Algorithm

TIES is a comprehensive and structured method to allow for the design of complex systems that are of high quality and competitive cost to meet future, aggressive customer requirements.[4] The method assists the designer by providing valuable insights and novel results in conceptual design phase to optimally direct program resources. The basic theory behind TIES method has been extensively explained by Mavris[5,6] and Kirby.[4,7] TIES provides a framework to create generic models to evaluate the impact of technologies on system level figures of merit. The models are in the form of a set of response surface equations, one for each objective. Technologies are modeled in the form of Technology Compatibility Matrix (TCM) and

Technology Impact Matrix (TIM). Interrelationship of technologies, i.e. incompatibilities and enabling, is represented by TCM, and TIM represents the impact of each technology on certain key parameters known as technology metrics ($k$-factors). A better understanding of the method is obtained from Figure 3. It shows the technology space $T$ mapped to the technology metrics or $k$-factor space via TIM; and the system model, in our example RSEs, is the function mapping $k$-factor space to the objective space. Hence, each technology combination from $T$ corresponds to a distinct objective vector in $R$. This formulation is called Technology Impact Forecasting and it is at the core of the TIES methodology. Once this environment is in place for a particular system, any number of technologies can be evaluated for that system, given the TCM and TIM for those technologies. A GA is wrapped around the technology impact forecasting environment to evaluate large number of technology combinations and select the best set of combinations. GA works by creating a pool of technology combinations and evaluating them in the technology impact model. This yields estimates of how technology combinations impact the system performance. The combinations are compared with each other and the best sets remain in the population pool for the next generation. The process is repeated for many generations until a global convergence is reached.

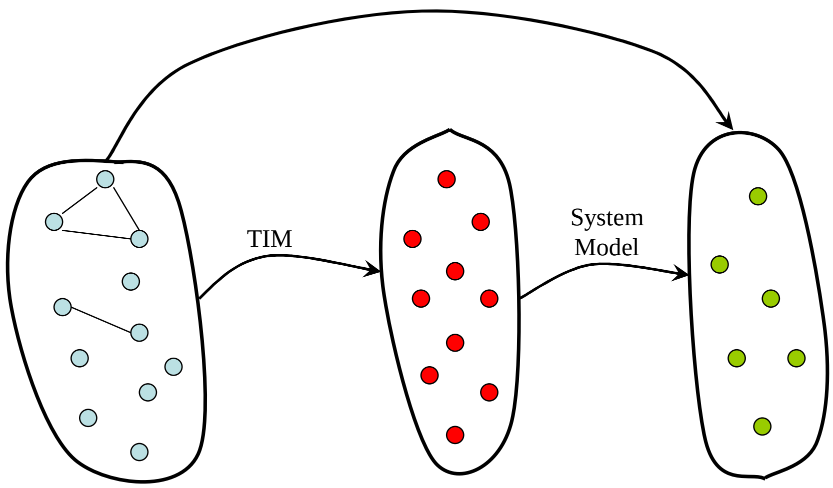


**Figure 3. Technology Combination Evaluation Model**

### B. Multi-Objective Decision Making and Pareto Optimization

The core problem at hand is of a Multi–Objective Decision Making. Given $t$ number of technologies and $n$ objectives, one has to decide on the best combination of technologies that satisfy all the requirements. MODM is associated with design problems; here, it is required to design the best alternative that satisfies all the constraints and meets all the requirements.[8] Hwang and Masud[8] classify various MODM methods based on the preference information from decision makers known before, during or after optimization process. These are stated below:

1. No Articulation of Preference Information: Here DM is not required to define any particular preference information after the problem is set up with constraints and objectives.

2. *A Priori* Articulation: Preference information is given by the DM to the analyst before solving the problem. This information can be in the form of a weighting or preference vector for the objectives.

3. *Progressive* Articulation: This is the class of interactive methods. Here, the DMs decide on their preferences based on the current solutions as the search progresses. There is a feed back loop between the DMs and analyst/machine.

4. *A Posteriori* Articulation: In this class of methods, MODM is divided into two distinct phases. In the first phase, a subset of non-dominated solutions in the objective space is determined. Next, the DMs make implicit tradeoffs between objectives based on some criteria, which may be non-quantifiable, and choose the most satisfactory solutions from the given subset.

The past implementation of GA for technology selection[2] belongs to the *a priori* class of MODM methods, where the DM articulates his preferences in the form of the weighting vector for objectives, before the optimization process. The focus of this paper is the *A Posteriori* class of MODM method where a subset of Pareto optimal technology solutions is searched and presented to the DM to make tradeoffs and select an appropriate solution.

### *1. Pareto–Optimal Solutions*

Pareto–optimal solutions are also termed as non–dominated solutions, efficient solutions, or non–inferior solutions in the literature.[8,9] In a Pareto–optimal solution set, no objective function can be improved without a simultaneous worsening in at least one of the other objectives. The concept of Pareto–optimal solutions and non–domination as applicable in NPGA is illustrated in Figure 4. The figure shows Pareto–optimal set of points $a, b, c$ and $d$ when two objectives $x$ and $y$ are to be minimized. The objective space is divided into four quadrants for evaluating the domination condition of point $e$. The lower left quadrant consists points that dominate $e$ and the upper right quadrant has points dominated by $e$. Other two quadrants are neutral, points in these quadrants have no bearing on the domination condition of $e$. The same logic can be extended to evaluate the domination condition of a point in $n$ dimensional space.

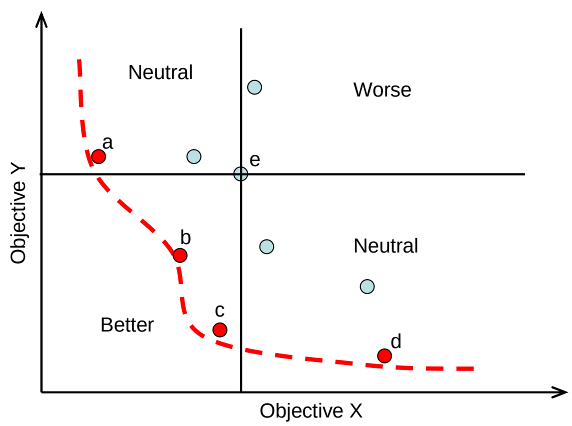


**Figure 4. Pareto–Optimal Solutions and Non–Domination (Adapted from [10])**

Given the dimensionality of the problem, searching for Pareto front solutions is a difficult task. Evolutionary algorithms are well suited for such problems; an extensive survey of evolutionary algorithms for Pareto optimization is given by Coello[11] and Veldhuizen[12] among several other researchers. Among the various Pareto optimization schemes using evolutionary algorithms, Niched Pareto GA is fast and simple to implement by changing just the reproduction operator of a canonical GA.

## II. Niched Pareto Genetic Algorithm

Niched Pareto Genetic Algorithm as proposed by Horn et. al.[13] is designed along the natural analogy of evolution of distinct species exploiting different *niches* or resources in the environment. In GA, niching is defined as the formation of subpopulations among large population set, with each subpopulation optimizing a subtask of the problem. A canonical GA is purely competitive where the best individuals quickly takeover the population. Whereas, when niching is included in the GA scheme, the populations tend to cooperate and the final set converges to a population of diverse species that are distributed along the Pareto frontier. The philosophy behind niching and NPGA has been discussed in detail by Horn.[9]

The basic implementation of NPGA concerns modifying the selection function of GA. One of the most widely used selection technique is *tournament* selection. Here, a subset of the population is randomly chosen and the best candidate in this set is selected. This implementation assumes that a single answer to the problem is desired. For NPGA, the selection method is modified to have multiple answers to the multi-objective problem. The selection method includes two main components: $(a)$ Pareto–domination tournaments, and $(b)$ Sharing.

### A. Pareto–Domination Tournaments

The tournament selection is altered to use multiple attributes for creating a Pareto frontier. To increase the domination pressure in the tournament selection, two *candidates* for selection and a comparison set of individuals is picked at random from the population. The number of individuals in the comparison set can be adjusted according to the requirements of the domination pressure. Each of the candidates is compared against each of the individuals in the comparison set. If one candidate is completely dominated by the comparison set and the other is not, then the later is selected. If both the candidates are dominated or non–dominated, then sharing is used to decide the winner.

### B. Sharing

Sharing helps to choose candidates when there is a tie after the tournament. If one of the candidates is randomly selected then, genetic drift will cause the population to group around in a single section of the Pareto front. To prevent this, equivalence class sharing in the objective space is implemented. Here, no preference is given to the two individuals regarding their objective values as they are already in the same equivalence class after the tournament. They are selected on the basis of density of population points in the neighborhood of a particular candidate. This density is calculated in the form of *niche count*, that is, the number of individuals present within the *niche radius* $\sigma_{sh}$ of a particular candidate. This parameter determines how far apart the individuals lie on the final Pareto frontier. The value of $\sigma_{sh}$ is under the control of the user and can be changed according to the requirements of a given problem. In order to determine $\sigma_{sh}$, Horn et. al.[13] suggest dividing the total surface area of the Pareto frontier with population size:

$$\sigma_{sh} \approx \frac{A_{pareto}}{N}$$

In this case, ideally, the population $N$ will be equally distributed, with $\sigma_{sh}$ units apart from one another, across the Pareto front. As for $A_{pareto}$, one may not know the exact area of the front but it is possible to determine the ranges of objective functions and with that, the range of $A_{pareto}$. With $M$ and $m$ denoting the vector of maximum and minimum magnitudes of objective functions respectively, for a two dimensional problem, $A_{pareto}$ will be greater than the hypotenuse given by:

$$A_{pareto} > A_{min} = \sqrt{(M_1 - m_1)^2 + (M_2 - m_2)^2}$$

The sum of the objective value ranges determines the upper bound for $A_{pareto}$:

$$A_{pareto} < A_{max} = (M_1 - m_1) + (M_2 - m_2)$$

In general, $A_{max}$ will be the sum of all the faces of a hyperparallelogram of edges $(M - m)$ determined by equation 1.[14]

$$A_{pareto} < A_{max} = \sum_{i=1}^{n} \prod_{\substack{j=1 \\ j \neq i}}^{n} (M_j - m_j) \tag{1}$$

It has been noticed that large difference in the magnitudes of various objectives can affect the distribution of population along the Pareto front.[9] This is due to the fact that Euclidian distance is used to measure the separation of two points on the Pareto frontier in $n$ dimensional objective space. This metric does not differentiate between the ranges and magnitudes of objective space. Hence one can have a skewed Pareto front if the objective values are used in the raw form. One of the most straightforward ways to avoid this kind of niching bias is to scale the objectives so that they are over the same magnitude.

# III. Aircraft Technology Selection Problem

The problem under consideration is of technology assessment for a 300 passenger commercial jet aircraft. There are 29 technologies to choose from and their impact is to be assessed on 15 system metrics or objectives. For convenience, technologies are denoted as T1, T2,. . ., T29 and objectives as R1, R2,. . ., R15. The objectives and some of the technologies considered are listed in Table 1 and Table 2 respectively. There are 60 technology metrics or $k$-factors on which the impact of technologies is mapped by the TIM, a part of which is illustrated in Table 3. The $k$-factors denoted by k1, k2,. . ., k60 usually are the multiplicative factors for lower level metrics such as compressor pressure ratio, turbine efficiency, sweep angle, component weights, etc. that are inputs to the system model. More details about TIM and $k$-factors is provided by Kirby.[4] It can be noticed that TIM is sparsely populated; this has considerable effect on the results and is explained in the following sections. The constraints on the solutions arise in the form of 11 incompatibilities among the technologies. Other conditions are also imposed on certain combinations of technologies.

**Table 1. Objectives Considered**

| Objectives | |
|---|---|
| R1 | L/D max M0.85 40,000ft (design) |
| R2 | Empty Weight of Aircraft Without Engines |
| R3 | Sideline Noise |
| R4 | Takeoff Noise |
| R5 | Approach Noise |
| R6 | Cruise TSFC |
| R7 | Thrust to Weight of Engine |
| R8 | NOx |
| R9 | econBlockFuel |
| R10 | TOGW |
| R11 | DOCi |
| R12 | LDGFL |
| R13 | TOFL |
| R14 | VAPP |
| R15 | ACQ |

**Table 2. Some of the Technologies Considered**

| Tech. No. | Technology Description |
|---|---|
| T1 | High Speed Slotted Wing |
| T2 | Transonic Adaptive Bump |
| T3 | Sensory Materials and Damage Science |
| T4 | ST Manufacturable Large Structures |
| T5 | Slat-Cover Filler |
| T6 | Landing-Gear Noise Reduction |
| T7 | Core Cowl Acoustic Liner |
| T8 | Installation Improved Chevron Nozzles |
| T9 | Flap Trailing Edge Treatment for Jet Interaction |
| ⋮ | ⋮ |

## A. Implementation

The NPGA is implemented for technology selection with canonical mutation and crossover operators. A fixed probability is assigned for mutation at 10% and crossover at 75%. The population is fixed at a

**Table 3. Section of Technology Impact Matrix**

| | k1 | k2 | k3 | k4 | k5 | k6 | k7 | k8 | k9 | k10 | k11 | k12 | k13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T12 | | | | | | | | | | | | | |
| T13 | | | | | | | | | 0.001 | | | | |
| T14 | | | | | | | | | 0.005 | | | | |
| T15 | | | | | | | | | | | | | |
| T16 | | | | | | | | | | | | | |
| T17 | | 1.475 | | 1.3 | | 25 | | | | 0.7 | 0.85 | | |
| T18 | | 1.475 | | 1.575 | | 25 | | | | 0.67 | 0.85 | | |
| T19 | | | | | | | | | | | 0.96 | -0.68585 | |
| T20 | | | | | | | | | | | 0.96 | | |
| T21 | | | | | -0.0166 | | 1.2812 | 1.9 | | | | | |
| T22 | | | | | | | | | | | | | |
| T23 | | | | | | | | | | | | | |
| T24 | | | | | | | | | | | | | |
| T25 | | | | | | | | | | | | | |
| T26 | | | | | | | | | | | | | |
| T27 | | | | | | | | | | | | | |
| T28 | | | | | | | | | | | 0.93 | -0.68585 | |
| T29 | | | | | | | | | | | | | 3650 |

relatively high number of 500 individuals to have a good distribution of points along the Pareto surface and the algorithm is executed through 150 generations. A *gene correction* algorithm is applied before the reproduction operator that changes the incompatible population members to compatible ones. This algorithm detects the incompatibilities within an individual and switches the technology states randomly so that the resulting individual has all compatible technologies. This technique is described in detail by Raczynski et. al.[15] The implementation of this algorithm guarantees that the NPGA results consist only the compatible combinations.

The reproduction operator that selects the population for the next generation has two parts as described before. The Pareto–domination tournaments is attempted first where a set of two *candidates* is compared against a set of 10 comparison individuals. All the 15 objective values of the two sets are compared to decide if any of the two candidates dominates the entire comparison set. If there is one non-dominated candidate, it is selected for the next generation. If both the candidates are either dominated by the comparison set or if both dominate the comparison set, equivalence class sharing in the objective space is implemented.

To accomplish this, the maximum and minimum value of each objective function in a population set is determined. From these values, the ranges for all the objectives for that particular generation is evaluated. Now a $n$-dimensional hypercube is formed around the two candidate points. The measure of each edge of this hypercube is equal to a certain multiple of $range/N$ in that particular dimension. Now, sharing is executed by counting the number of population points that are present within the hypercube around each candidate points. The candidate with minimum number of neighbors represents a sparse region on the Pareto surface and is selected for next generation. This arrangement for sharing eliminates the need for specifying $\sigma_{sh}$ and also to measure the Euclidian distance to perform sharing. As the hypercube constructed for sharing has dimensions relative to the objective values, the need to scale the objectives is avoided. Another advantage of this approach is that the dimensions of hypercube constructed are dynamic in nature and change from generation to generation; it is more representative of the nature of current population.

## B. Results

Running on a 2 Mhz Pentium-4 PC, the NPGA takes about 11-12 minutes. The primary results are in the form of the final population of technology combinations and corresponding objective values. This set of individuals in the form of $N \times t$ and $N \times n$ matrices is then transferred to a statistical and visualization software by SAS Institute called JMP$^{\text{TM}}$.[16] A screen capture of this environment is shown in Figure 5. The bottom window in this figure shows a table with first 29 columns representing the on/off conditions for 29 technologies and the next 15 columns represent the objectives. The rows represent population members of the final generation. The upper left window is for a three dimensional rotatable plot. Here, one can select any three objectives of interest and view the resulting three dimensional plot by rotating it at different angles. A draftsman's plot, also known as a plot matrix is depicted in the upper right window of the figure. It consists of plots of the values for each objective against the values for each of the other objective. One can select any number of objectives for draftsman's plot; first five objectives are selected in the example depicted in the figure. This serves as a basic tradeoff environment where DMs can view the data table and plots simultaneously. They can select an individual point or group of points in any of the plots and view the technology combination and response values corresponding to that point in the table. They can view the location of that point with reference to any of the selected axis in the spinning plot or draftsman's plot. The screen capture in Figure 5 shows a point selected on the plots corresponding to row 28. The three dimensional plot and the draftsman's plot with point 28 highlighted are shown in Figure 6 on the following page and Figure 7 on the next page respectively.

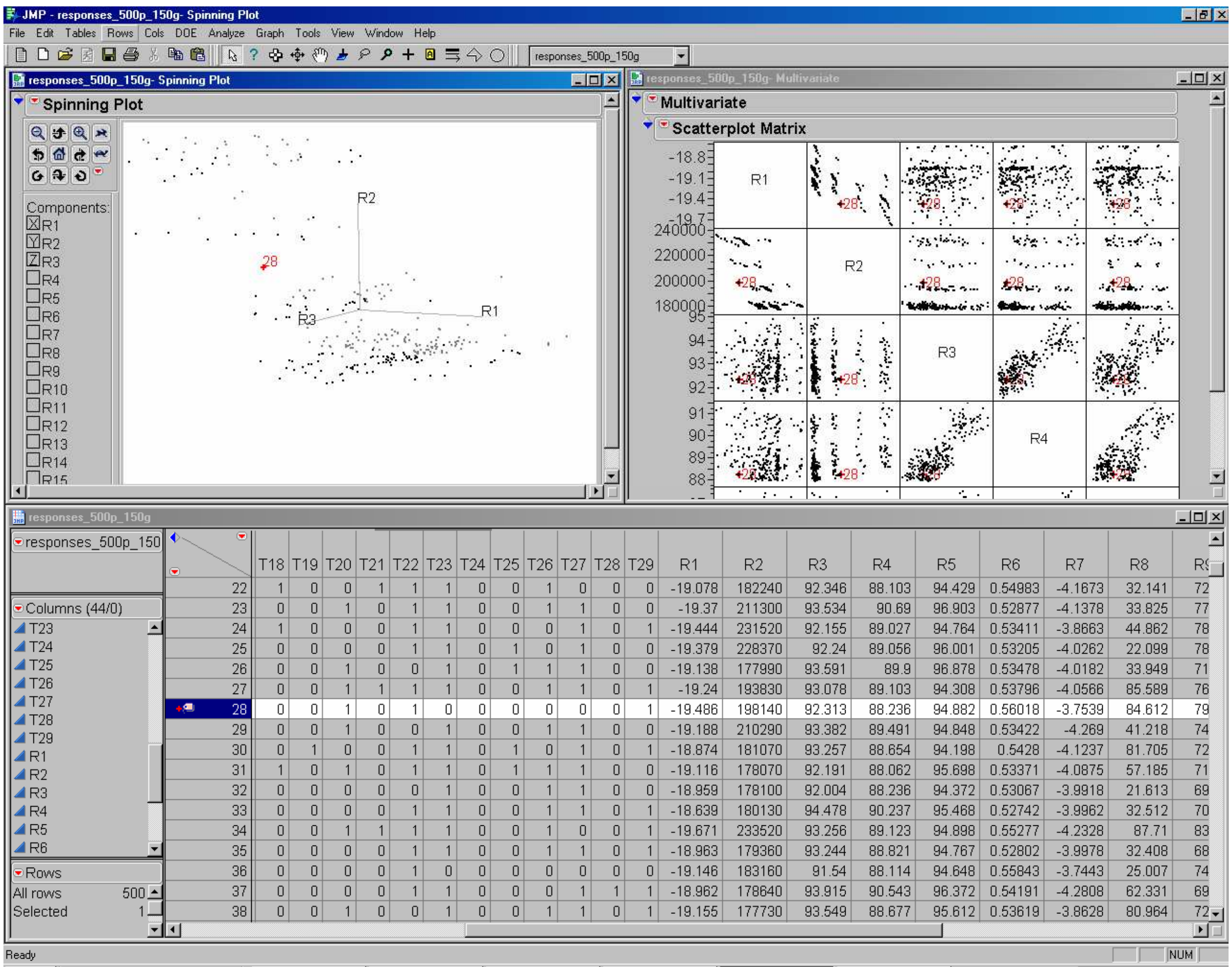


**Figure 5. JMP$^{\text{TM}}$ environment**

Frequency of occurrence of each technology in the final generation is computed and a plot showing the frequency on vertical axis and technology number on horizontal axis is shown in Figure 8 on the following page. As evident from the figure, the technologies are more or less evenly distributed in the final generation. The only definite conclusion that can be reached from this figure is to exclude technologies like T11, T15, T24 and T28 that are present in very few members of the final population. Thus, the Pareto–frontier has

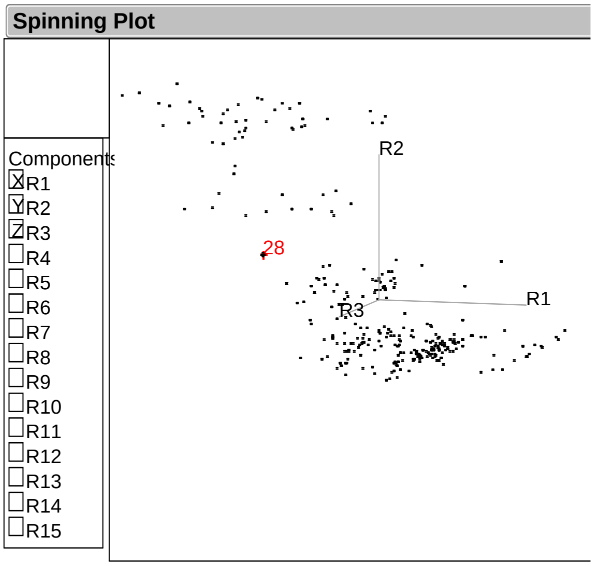


**Figure 6. Three Dimensional Spinning Plot**

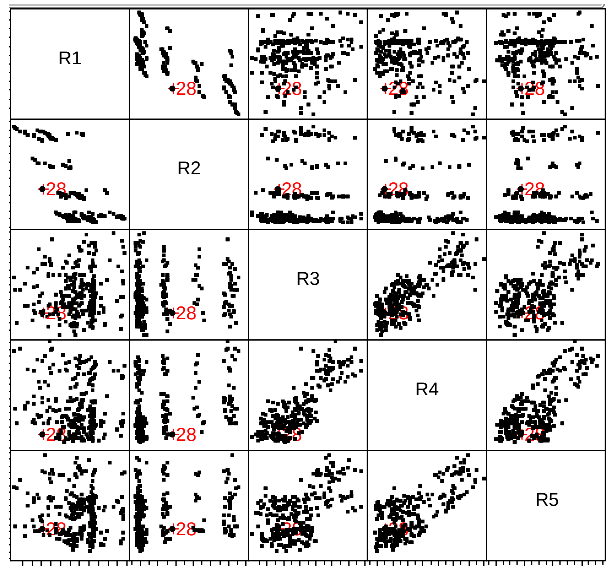


**Figure 7. Draftsman's Plot**

almost all technologies switched on in one combination or another; i.e., apart from the above mentioned four, all other technologies are active on the Pareto–frontier. This type of plot gives the DM an idea of the composition of technologies on the frontier. If only few technologies are active on the frontier, the decision making becomes much easier. Unique technology combinations present in the final generation are identified and plotted in Figure 9. This figure illustrates there are 295 unique technology combinations out of 500 individuals. These unique individuals are evenly distributed, i.e. none of the combinations dominate the populations and none of the individuals are repeated more than 5 times.

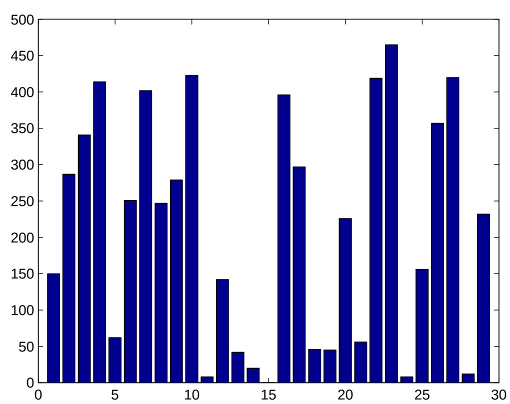

**Figure 8. Frequency of Occurrence of Each Technology in Final Generation**

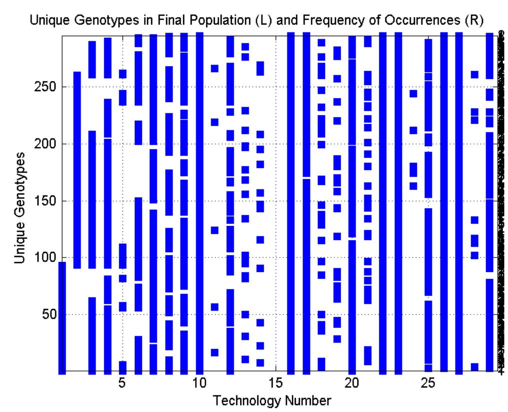


**Figure 9. Unique Technology Combinations in Final Generation**

*1. Extracting Pareto Frontier*

NPGA being a stochastic algorithm, its final generation cannot be expected to be composed of all non–dominated solutions. It is possible that few points are dominated by others in the final set. To extract the Pareto solutions and discard dominated ones, an efficient sorting algorithm is implemented along the lines of one described by Deb et. al.[17] To decide if a solution is non–dominated, it has to be compared with all other points in the set. That is, an algorithm will need $N(N-1)/2$ comparisons to find non–dominated solutions from a set of $N$ candidates. The algorithm used here finds the non–dominated solutions with considerably less comparisons, the exact number of which depends on the number of dominated points present in the set. For the given problem, 287 non–dominated points are extracted from the final population of 500 individuals. This number is almost equal to the number of unique combinations, i.e. 295, as mentioned before. This shows, only $295 - 287 = 8$ unique individuals are dominated by others members in the final population and thus, most of the final individuals of NPGA belong to the Pareto–frontier. The number of comparisons required is 62,562 instead of 124,750 if each individual was compared with the other. This algorithm can also be used to extract Pareto solutions belonging to a subspace of the entire objective space. A plot depicting Pareto solutions for response R1 and R2 is illustrated in Figure 10. There are 26 non–dominated solutions, in the objective space of R1 and R2, present in the final population. The R1 axis has negative values because the algorithm is set up for minimization and R1 i.e., L/D max has to be maximized. It can be observed that by moving from right to left on R1 axis towards higher L/D values, the empty weight of the aircraft (R2) increases. That is to say, there is a weight penalty for higher L/D max. These two or three dimensional Pareto plots are useful for DMs to visualize the Pareto–frontier solutions in the objective space of maximum interest.

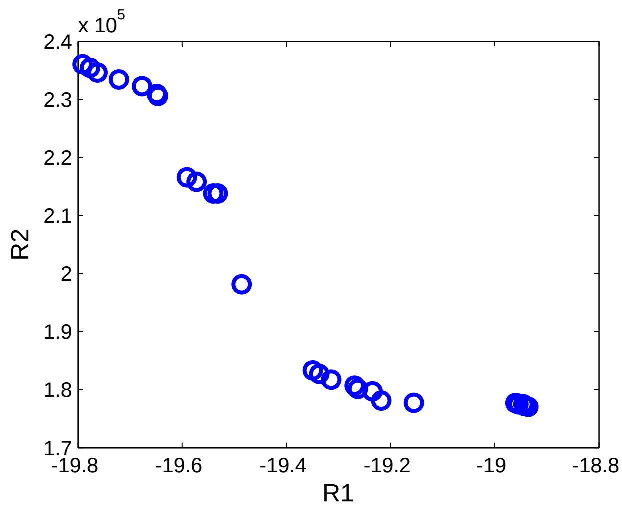


**Figure 10. Pareto Solutions in Two Dimensional Space of R1 and R2**

It can be noticed in Figure 7 on the preceding page and Figure 10 that the solutions tend to form clusters or plateaus in the objective space. This phenomenon can be attributed to the fact that this is a combinatorial problem. As certain groups of technologies are switched on and off, there is a discrete change in the responses. It is observed that each of these clusters have certain technologies that are consistently on or off. The sparsity of TIM also contributes towards this behavior. When TIM is densely populated, the Pareto solutions are more likely to be evenly distributed in the objective space.

## IV. Conclusion and Future Research

Results from this study suggest that the *A Posteriori* method based on NPGA can be a powerful tool for technology selection. Here the DMs do not have to make decisions in information vacuum. They are aware of the entire solution space and can make tradeoffs accordingly. This method divides the work between technology analysts and decision makers; the analyst sets up the problem, performs optimization and presents the Pareto-optimal solution set to the DMs. As a result DMs can arrive to satisfactory solutions efficiently without spending too much time. They can make tradeoffs based on important criteria that often are not quantifiable.

One of the drawbacks of this method in its present form is the exponential increase in number of Pareto-optimal solutions with increase in the dimension of objective space. This creates difficulties for DMs to arrive at the "right" solution as it is not possible to clearly visualize solutions in more than 3 dimensions. This difficulty can be alleviated to some extent by using some form of Multi Attribute Decision Making (MADM) technique for the final selection. MADM methods are applicable where there is a set of probable solutions and the DM has to select from that set. Hence, unlike MODM, MADM only deals with selection and not design of the best alternative. Next logical step in this direction would be to create a decision making environment that brings results from consistent analytical foundation, in form of Pareto–solutions, in a dynamic environment where the DMs can carry out MADM studies and interactively select technologies. The benefits of this method can be further exploited when a higher fidelity model is used in place of RSEs as used in this application. This will result in more accurate results without any significant increase in time and effort required on the part of DMs to select the solutions.

## V. Acknowledgements

We gratefully acknowledge the support of NASA Glen Research Center under the task order number NNL04AD06T of contract NAS 3-00179 . The authors would also like to thank Mr. Carl Johnson for his contribution to this work.

## References

[1]Roth, B., German, B. J., Mavris, D. N., and Macsotai, N., "Adaptive Selection of Engine Technology Solution Sets From a Large Combinatorial Space," AIAA Paper 2001–3208, 2001.

[2]Roth, B. and Patel, C., "Application of Genetic Algorithms in the Engine Technology Selection Process," *Journal of Engineering for Gas Turbines and Power*, Vol. 126, No. 4, 2004, pp. 693–700, also ASME Paper 2003–GT–38482, June 2003.

[3]Roth, B., Mavris, D. N., Graham, M. D., and Macsotai, N., "Adaptive Selection of Pareto-Optimal Engine Technology Solution Sets," ICAS Paper 2002–594, 2002.

[4]Kirby, M. R., *A Method for Technology Identification, Evaluation and Selection in Conceptual and Preliminary Aircraft Design*, Ph.D. thesis, Georgia Institute of Technology, Atlanta, GA 30332, 2001.

[5]Mavris, D. N., Kirby, M. R., and Qui, S., "Technology Impact Forecasting for a High Speed Civil Transport," SAE Paper 985547, 1998.

[6]Mavris, D. N. and Kirby, M. R., "Technology Identification, Evaluation and Selection for Commercial Transport Aircraft," SAWE Paper 2456, Category no. 11, 1999.

[7]Kirby, M. R. and Mavris, D. N., "Forecasting Technology Uncertainty in Preliminary Aircraft Design," SAE Paper 1999–01–5631, 1999.

[8]Hwang, C.-L. and Masud, A. S. M., *Multiple Objective Decision Making - Methods and Applications*, No. 164 in Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Birlin Heidelberg New York, 1979, In Collaboration with Sudhakar P. Paidy and Kwangsun Yoon.

[9]Horn, J., *The Nature of Niching: Genetic Algorithms and the Evolution of Optimal, Cooperative Populations*, Ph.D. thesis, University of Illinois at Urbana-Champaign, Urbana, IL 61801, 1997.

[10]Zitzler, E., "Evolutionary Algorithms for Multiobjective Optimization," *Evolutionary Methods for Design, Optimisation and Control*, edited by K. Giannakoglou, D. Tsahalis, J. Periaux, K. Papailiou, and T. Fogarty, CIMNE, Barcelona, Spain, 2002.

[11]Coello, C. A. C., "Evolutionary Multi-Objective Optimization: A Critical Review," *Evolutionary Optimization*, edited by R. Sarker, M. Mohammadian, and X. Yao, International Series in Operations Research and Management Science, chap. 5, Kluwer Academic Publishers, 2002.

[12]Veldhuizen, D. A. V. and Lamont, G. B., "Multiobjective Evolutionary Algorithms: Analyzing the State-of-the-Art," *Evolutionary Computation*, Vol. 8, No. 2, 2000, pp. 125–147.

[13]Horn, J., Nafpliotis, N., and Goldberg, D. E., "Multiobjective Optimization Using The Niched Pareto Genetic Algorithm," IlliGAL Report 93005, University of Illinois at Urbana-Champaign, Urbana, IL 61801, July 1993, also, Published, in part, in The Proceedings of the First IEEE Conference on Evolutionary Computation, IEEE World Congress on Computational Intelligence, Volume 1, 1994, pp. 82–87.

[14]Fonseca, C. M. and Fleming, P. J., "Genetic Algorithms for Multiobjective Optimizations: Formulation, Discussion and Generalization," *Proceedings of the Fifth International Conference on Genetic Algorithms*, edited by S. Forrest, Morgan Kaufmann, San Mateo, CA, July 1993, pp. 416–423.

[15]Raczynski, C. M., Lewe, J.-H., Kirby, M. R., and Mavris, D. N., "Technology Portfolio Assessments Using a Gene-Correction Genetic Algorithm," AIAA Paper 2003–6731, 2003.

[16]"JMP Computer Program and Users Manual," SAS Institute, 2005.

[17]Deb, K., Agrawal, S., Pratab, A., and Meyarivan, T., "A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II," *Proceedings of the Parallel Problem Solving from Nature VI Conference*, edited by M. Schoenauer, K. Deb, G. Rudolph, X. Yao, E. Lutton, J. J. Merelo, and H.-P. Schwefel, Springer. Lecture Notes in Computer Science No. 1917, Paris, France, 2000, pp. 849–858.